Int. Agrophysics, 1994, 8, 597-601

MODELLING THE DEFORMATION OF CELL TO LOADS USING MODEL CELL OF RUBBER BALL FILLED WITH WATER

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Abstract. Physical property of plant has to be described on the basis of cellular structure. However, even deformation of a single cell has not been confirmed experimentally. In this study, deformation of single cell is analysed. Following four conditions are established for plant cell: 1) the cell wall can be treated as isotropic elastic membrane, 2) the cell fluid is incompressible, 3) effect due to exosmosis during compression is neglected and 4) the deformation due to mass of cell fluid is ignored.

Under these conditions, single cell under compression is supposed to change from sphere to ellipsoid of revolution and can be modelled with rubber ball filled with water. Then relation among volume, stress, strain and pressure were analysed as ellipsoide. Parameters of ellipsoid were also identified in experiment using rubbur ball. Calculated value were fitted to measured value. The results, confirmed that model cell was changed from sphere to ellipsoid of revolution.

Keywords: deformed cell, rubber ball, ellipsoid of revolution, uniaxial test

INTRODUCTION

Plant which consists of cell is not continuum. Therefore, physical property of plant has to be described as discontinuous body on the basis on cellular structure. Pitt [2-6] reported several papers following such way of approach. In case cell tissues are fulled up like potato, it is said that cell shape becomes tetrakaidecahedron. Gao and Pitt [1], also, reported paper to deal with cell shape as a tetrakaidecahedron. However, cell mechanics is a developing science, now, and even deformation of a single cell is not confirmed experimentally. Cell shape of inner part of watermelon is not mature in comparison with potato, and the shape is similar to elliptical single cell.

For this reason, deformation of single cell can be analysed as first step of analysis of cell conglomerate. As to deformation analysis of cell, following condition are usually assumed: 1) the cell wall can be treated as isotropic elastic membrane, 2) the cell fluid is incompressible, 3) effect due to exosmosis during compression is neglected and 4) the deformation due to mass of cell fluid is ignored.

Assumptions 1) and 3) must be confirmed in experiment. If these assumptions were established as conditions in analysis, single cell is modelled by a rubber ball filled with water.

In this study, these assumptions are confirmed in experiment, and deformation of single cell is analysed. Then experiments with model cell were performed.

MODELLING OF SINGLE CELL UNDER COMPRESSION

Modelling is performed under above-mentioned four assumptions. As shape of free single cell is a sphere, external form on meridional plane is a circle. If the circle compressed continuously, it seems that the circle becames an ellipse. Since shape of cell is kept axisymmetric under uniaxle compression, cross sections parallel to equator are kept circle. Therefore, the shape of single cell under compression' is changed from sphere to ellipsoid of revolution as shown in Fig. 1. Equation of ellipse is given by:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 , \quad x = f(y) . \tag{1}$$

Volume of ellipsoid as deformed cell is given by:

$$V_e = 2\pi \int_0^h f(y)\, dy = 2\pi a^2 h \left(1 - \frac{h^2}{3b^2}\right) \tag{2}$$

where $2h$ is height of the deformed cell. Volume of original sphere is given by:

$$V_s = \frac{4}{3} \pi r^3 \tag{3}$$

where r is radius of sphere. As cell fluid is incompressible, Ve is equal to V_s. However, the surface to volume ratio increases under compression. Thus the cell surface area must expand, and stress of the cell wall is increased. If bending of the wall is ignored provided thickness of wall t is thin enough compared with the radius of curvature, equilibrium equation at a given point is given by:

$$\frac{\sigma_m}{r_m} + \frac{\sigma_h}{r_h} = \frac{p}{t} \tag{4}$$

where σ_m and σ_h are meridional and hoop stress, respectively; r_m and r_h are meridional and hoop radius of curvature, respectively; p is turgor presssure; t is thickness of cell wall. Radius of curvature of ellipse is given by:

$$r = \left| \frac{\left\{a^4 + (a^2 - b^2) x^2\right\}^{3/2}}{-a^4 b} \right| \tag{5}$$

r_h is determined by following procedure. Equation of normal at point (x_1, y_1) is given by:

$$y - y_1 = [dx/dy]\, x = x_1 (x - x_1) \tag{6}$$

Cross section which ellipsoid of revolution is cut by perpendicular plane, including this normal, to $x - y$ plane become ellipses as shown in Fig. 1, too. Radius of curvature at a point (x_1, y_1) which the normal intersects the ellipse is equal to r_h. Therefore, if parameters of the ellipsoid of revolution are known, r_m and r_h at given point is determined.

Constitutive equations of istropic elasticity are given by:

$$\begin{aligned} \varepsilon_m &= (\sigma_m - v\sigma_h) / E \\ \varepsilon_h &= (\sigma_h - v\sigma_m) / E \end{aligned} \tag{7}$$

where ε_m and ε_h are meridional and hoop strain, respectively. E is modulus of elasticity, v is Poisson's ratio; strain ε_m and ε_h are calculated from peripheral length of circle before deformation and ellipse under compression.

MATERIAL AND METHOD

Confirmation of conditions in analysis

In plant under fixed displacement, stress relaxation occurrs due to plastic deformation of cell wall or exosmos of cell fluid. If the stress relaxation rate is far lower than intended uniaxle compression rate, assumption 1) and 3) are established approximately as conditions in analysis. Cells of watermelon is supposed in this study, and the relaxation rate was measured with cylindrical sample of inner

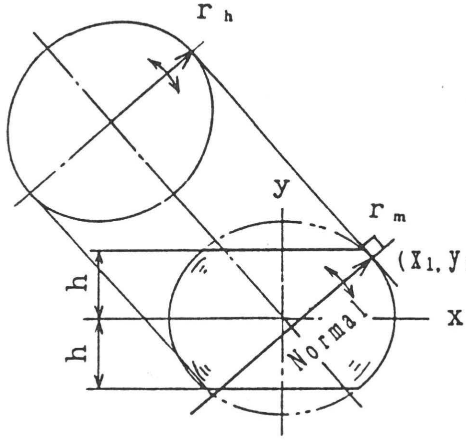

Fig. 1. Radius of curvature of deformed single cell.

part of watermelon. Also, cell rearrangement may be occurred by debonding and disappearance of intercellular void under compression. If the response under cyclic loading can be observed, effect of rearrangement to assumptions can be confirmed.

Small material tester (Rheometer Model CR-200D, Sun Scientific Co.) was used. Cylindrical samples, diameter 14.5 mm and 15 mm height, were compressed at rate of 0.167 mm/s (10 mm/min). In cyclic loading, the push rod of the tester was given displacement from the point at which the sensor received a set value of reactive force. If plastic deformation of specimen is occurred during the test, absolute acting range of the rod was changed though the displacement of the rod would be constant. Results at rod acting range of 3 mm is shown in Fig. 2. It shows that inclination of the first cycle is different from other. However, inclinations of other cycle are similar, although maximum forces are different because of the rod acting range. As the result, most intercellular void disappeared under first compression. Therefore, if stress relaxation test was performed after about four compression cycles, effect of intercellular void can be eliminated.

The relaxation rate was about 0.02 N/min. as shown in Fig. 3. If compressive rate is far more than this rate, assumption 1) and 3) are established as conditions in analysis. Therefore, single cell subjected to such a condition like cell of inner part of watermelon can be modelled by a rubber ball filled with water.

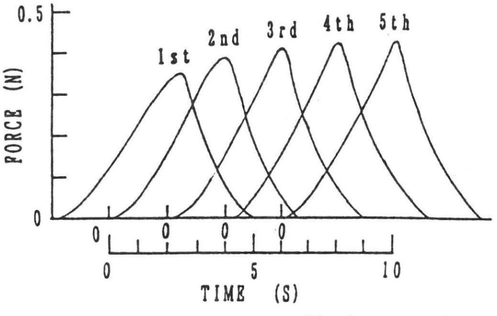

Fig. 2. Force under cyclic loading.

Experiment of model cell under compression

Measured items and instruments are shown in Table 1. In laser displacement transducer, 1 mm thickness and 10 mm width laser beam is emitted from projector. As the deformed ball interrupt the laser beam, lateral displacement of the ball can be measured by laser beam width received sensor head, i.e., output voltage. The model cell was filled with water at inner pressure of 5 kPa. Compression rate was 0.13 mm/s (8 mm/min). Model cell was made of rubber ball which characteristics are as follows: modulus of elasticity E=800 kPa, Poisson's ratio $\gamma=0.45$ and thickness t=1.0 mm. Material testing device (UTM-4 Toyo Keiki Co.) like Instron was used for uniaxle compression test of model cell as shown in Fig. 4.

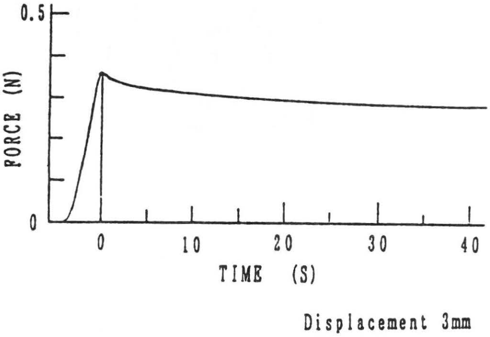

Fig. 3. Stress relaxation.

Table 1. Measured items and instrumentation

Measured items	Instrumentation
Inner pressure	Strain gauge type pressure gauge
Force of cross head	Load cell
Vertical displacement (cross head and piston)	Differential transformer
Lateral displacement	Semiconductor laser displacement transducer (Type H, Keyence Co.) (Wave length 780 nm, Output power 3 mW)

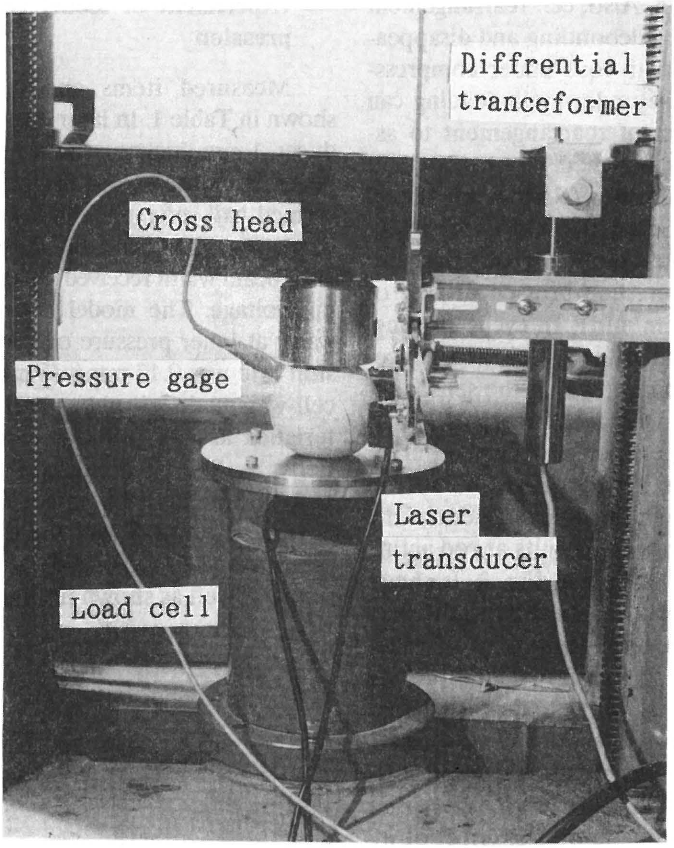

Fig. 4. Compression test with model cell.

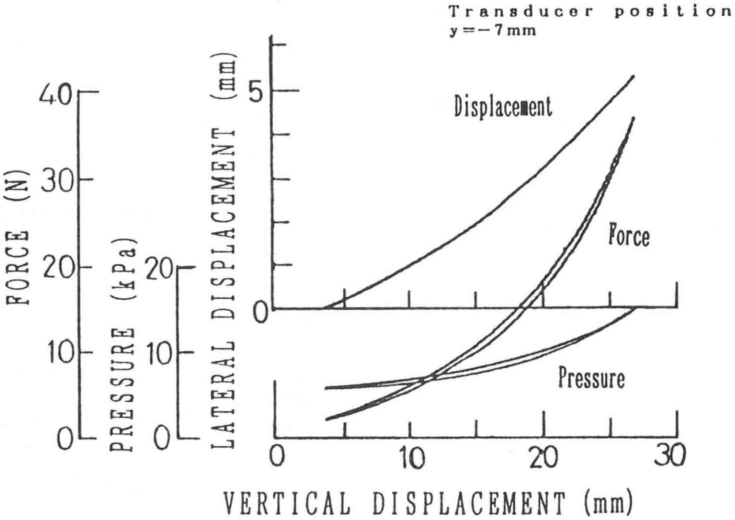

Fig. 5. Lateral displacement, pressure and force.

RESULTS AND DISCUSSION

Lateral displacement, inner pressure and force of load cell are shown in Fig. 5 provided the displacemelnt was measured at the position of y=-7 mm. From these data, parameters of ellipse *a* and *b* were identified with least square method. Volume and pressure were calculated using identified parameters from Eqs (1)-(7). The result including radius of curvature on equater is shown in Table 2.

The table shows that calculated pressure fits to measured value. In this result, it is verified that the rubber ball in neighbourn of equator become ellipsoid of revolution. Furthermore, taking into account measured error, it is confirmed that volume of ball did not change.

Table 2. Comparison calculated and measured inner pressure

Displacement (mm)		Parameter of ellipse (mm)		Radius (mm)	Volume (cm^3)	Stress (kPa)		Pressure (kPa)	
								calculated	measured
y_c	h	a	b	r_m	V_e	σ_m	σ_h	P	p
0	33.0	33.0	33.0	33.0	75.3	0.0	0.0	5.0	5.0
5	30.5	33.1	33.3	33.5	75.6	3.0	1.8	5.2	6.1
10	28.0	33.6	33.0	32.4	75.5	18.3	15.8	6.0	6.9
15	25.5	34.3	34.4	34.5	77.0	43.8	8.8	6.5	8.3
20	23.0	35.5	33.8	32.2	77.0	76.0	74.8	9.7	10.4
25	20.5	37.3	31.3	26.3	76.8	152.5	107.0	13.2	13.5

y_c - cross head displacement, r_m, σ_m, σ_h - values on the equator, r_h on the equator is equal to parameter a.

CONCLUSIONS

Physical property of plant has to be described on the basis of cellular structure. In single cell, a following four conditions are established: 1) the cell wall is an isotropic elastic membrane, 2) the cell fluid is incompressible, 3) effect due to exosmosis during compression is neglected, 4) the deformation due to mass of cell fluid is ignored. Assumptions 1) and 2) were confirmed in the experiment. Under these conditions, it was indicated that single cell under compression was changed from sphere to ellipsoid of revolution and can be modelled with rubber ball filled with water. Then, relation among volume, stress, strain and pressure were analysed as ellipsoid. Parameters of ellipsoid were identified in experiment using rubber ball. Volume and inner pressure were calculated with this parameter. Calculated value fitted to measured value. Therefore, it is supposed that actual single cell under compression become from sphere to ellipsoid of revolution.

This study is the first step to develop cell mechanics. We would like to analyse and experiment on cellular conglomerate from now on.

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